Abstract—To determine a force-closure grasp, current grasp synthesis algorithms either assume a deterministic model to compute a desired finger placement without noise, or model the end-effector position as a free-floating rigid body whose noise in pose is independent of the kinematic chain formed by the robot arm. In this work, we instead explore a probabilistic approach that explicitly models noise in joint-angles. By sampling additive noise that is applied to a pre-grasp configuration and studying the resulting probability of force-closure when the robot fingers are closed, we observe in experiments that joint-angle positions can have a remarkable effect on the probability of successfully restraining an object. We systematically study the grasp quality value as a random variable and investigate the convergence of sampling-based estimators for the mean, covariance and moments up to third order of this quantity by means of Monte Carlo Sampling. We study illustrative examples of the impact of initial joint-configurations on the likelihood of force closure on a seven degree of freedom simulated Kuka lightweight robot arm.

I. INTRODUCTION

The study of grasp quality metrics is deeply intertwined with the study of grasp planning algorithms. This connection stems from the fact that in most cases, a robotic manipulator can grasp a given object in multiple ways. Given a set of possible solutions determined by the grasp planner, it is then natural to ask which one should be preferred, and the utility of grasp quality metrics is therefore evident. While many grasp quality metrics have been proposed [5], [10], [17], [21], most contributions have ignored the morphology of the robot executing the grasp. For example, the Ferrari-Canny metric [5] emerged as the most used metric, but it just considers the contact points without incorporating kinematic constraints. Most other methods embrace a similar standpoint [10], [17], [21]. Moreover, most classical grasp quality metrics do not incorporate a fully realistic noise-model accounting for the inevitable inaccuracies occurring when a grasp is executed. This problem is becoming particularly relevant with the advent of platforms with passive joints (e.g., Baxter [6]). For example, our experiments show that the grasp shown on the left has a significantly lower probability of force closure and expected Ferrari-Canny grasp quality compared to the grasp on the right. The same results are observed throughout the numerous experiments we present in this manuscript. We argue that noise-models that fully incorporate the kinematic structure and limitations of a robot should be used to more reliably predict the success of a proposed grasp. The main goal of the present paper is to highlight the importance of noise in joint-configurations for the purpose of grasp quality evaluation and synthesis. While error propagation is a well-studied problem in mechanics and robot arms [4], the impact of errors in joint-angles has, to the best of our knowledge, at this point not been integrated with the main-stream robotic grasp quality evaluation literature.

In this work, we propose to explicitly account for noise in joint-angle positions, and to study the relationship between this noise and the definition of the grasp quality metric $Q$ defined by Ferrari and Canny. We focus in particular on the probability of force closure measure $P(FC)$ [13] and the expected grasp quality $E[Q]$. The reader is referred to Figure 1 to consider the motivations for this work. It shows an example of two grasps with identical contact point configurations. Note however the relative rotation of the third joint by $\frac{\pi}{2}$. When imposing Gaussian noise on the individual joints of the robot arm, the covariance ellipsoid of end-effector positions is also rotated by $\frac{\pi}{2}$, resulting in a drastically changed value for $P(FC)$.

Fig. 1: The above figures display two grasps with identical contact point configurations. Note however the relative rotation of the third joint by $\frac{\pi}{2}$. When imposing Gaussian noise on the individual joints of the robot arm, the covariance ellipsoid of end-effector positions is also rotated by $\frac{\pi}{2}$, resulting in a drastically changed value for $P(FC)$.

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ology sustaining our study. Simulations and their results are illustrated in Section IV, while conclusions and future work are discussed in Section V.

II. BACKGROUND AND RELATED WORK

A. Analytic Grasp Synthesis

Over the last two decades various grasp quality metrics have been developed with the aim of turning the grasp synthesis problem into an optimization problem. The reader is referred to [2], [18] for surveys of these methods, in particular for aspects related to physics-based models, such as the quasi-static model [15] to derive force or form-closure grasps. The Ferrari-Canny metric [5] ranks grasps based on their ability to resist an arbitrary disturbance wrench acting on the object being restrained. This metric has gained significant popularity despite the fact that it is suffers from various drawbacks, i.e., it is not scale invariant, and its numerical value depends on an arbitrary choice of the point with respect to which torques are computed. Strandberg and Wahlberg [22] proposed an alternative metric that overcomes some of these limitations and in particular considers only disturbance wrenches that may occur in practice. This measure, however, has been scarcely used in practice because of its computational burden. Recently, Liu and Carpin have proposed two methods based on partial quick hull computation to significantly accelerate the computation of these metrics [11], [12]. A recent survey of grasp quality metrics is given in [19]. The above mentioned methods do not model noise in the grasping process and are based on analytic physics based approaches requiring perfect knowledge of numerous parameters, including friction coefficients, contact points and surface normals. In most instances, these values are instead only known with uncertainty.

Data-Driven Approaches

An alternative approach to analytics grasp synthesis is given by data-driven approaches, where grasps are obtained combining prior data to learn how to grasp a new object. The reader is referred to [3], [9] and references therein for recent results obtained with this class of methods. With data-driven approaches, noise is implicitly accounted for when data is gathered, but it is often not explicitly modeled. Therefore, the ability to predict how a grasp configuration will be affected by a change in the noise is limited and usually not explicitly accounted for.

Noise modeling in mechanics

As pointed out in [16], noise in modeling robot arm mechanics derives from four sources, i.e., 1) deterministic inaccuracies in the geometric models of links; 2) nondeterministic geometric errors due to backlash and dimensional changes due to compliance; 3) nondeterministic errors due to friction, and external loads; 4) quantization errors in sensing and control. Most papers studying these noise sources embrace a linearized error propagation models and Gaussian distributions, although papers like [16] model errors starting from the empirical error distribution. Coupled with error modeling, there is also a rich literature parameter identification using either analytic approaches [7] or machine learning techniques [1], [23].

Noise modeling for grasping

Noise in grasp contact points has so far been considered only independently of the kinematic structure, for example in the work of [8], where Gaussian errors in the end-effector positions, friction coefficient and object shape lead to a notion of probability of force closure. Similarly, uncertainty in the object parameters led to the notion of probabilistic force closure [24] and has been applied in [13], were the authors proposed a large-scale cloud-based approach to sampling perturbations of grasps and leverages multi-armed bandits and deep learning to determine grasps with high probability of force closure. The approach taken in the present paper is instead to consider the effect of noise on joint-angles on the final grasp quality.

III. PROBLEM DEFINITION AND METHODOLOGY

Problem Definition

Consider a robot arm with \(d\) degrees of freedom. Let \(f : \mathbb{R}^d \to \text{SE}(3)\) denote the forward-kinematics function mapping a joint-configuration \(q = (q_1, \ldots, q_d) \in \mathbb{R}^d\) to the position and orientation of a coordinate system placed at the end-effector, e.g., in the palm of the robot hand. A typical approach to grasp planning is to determine a pose and orientation of the hand base that results in a stable grasp when the fingers of the robot hand are “autoclosed” sequentially and stopped when they touch the object. This approach is used for example in the popular grasp planner GraspIT [14]. The quality of the resulting grasp can then be ranked with a grasp quality metric \(Q\) associating a real valued score to the contact points. In this work, we assume that the joint-angles of the robot can be set to a target value \(q_0\) only up to some additive noise \(\varepsilon = (\varepsilon_1, \ldots, \varepsilon_d)\) where the \(\varepsilon_i\) are independent random variables following a known distribution, e.g., Gaussian, uniform, or triangular. Without loss of generality, we assume that each \(\varepsilon_i\) is zero-mean. We denote by \(N(0, \sigma^2)\) the univariate Gaussian distribution with mean 0 and variance \(\sigma^2\). From this assumption, it follows that when the robot is commanded to achieve a target configuration \(q_0\) it will instead reach a configuration \(q = q_0 + \varepsilon\). As a result, the grasp quality score \(Q\) under noise is a random variable, and we aim at studying its dependence on \(\varepsilon\) and the kinematics of the robot arm. We in particular consider the Ferrari-Canny metric which measures the size of the largest wrench that can be resisted in all directions by the grasp. If the grasp fails to be in force closure, or if contact is not established, the grasp quality metric is not defined. Therefore another binary random variable is implicitly defined, i.e., the variable indicating whether a grasp is force closure or not. The probability of this variable being equal to 1 is the probability of force closure \(P(FC)\) we mentioned earlier. To study the properties of \(Q\) (as a random variable) and \(P(FC)\), we investigate the mean, variance and the third standardized moment, i.e., the skewness. The goal
of our analysis is to identify differences in grasp robustness caused by variations in arm configurations, to enable a robot to choose an optimally robust grasp under noise in kinematics.

**Sampling based approximation**

Given a target grasp configuration \( q_0 \) determined for example with a grasp planner such as GraspIT, we study the resulting grasp quality of the random variable \( Q(f(q_0 + \varepsilon)) \) where we explicitly outline the dependency of \( Q \) on both the forward kinematics map \( f \) and the noise \( \varepsilon \). To this end, we generate a finite number of samples \( \varepsilon^1, \ldots, \varepsilon^n \in \mathbb{R}^d \) and let \( X_i = Q(f(q_0 + \varepsilon^i)) \). We then compute the \( n \)-sample empirical estimators (mean, standard deviation, co-variance, skewness):

\[
\mathbb{E}[Q, q_0] \approx \frac{1}{n} \sum_{i=1}^{n} X_i = \overline{Q}
\]

\[
\sigma \approx \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (X_i - \overline{Q})^2}
\]

\[
\text{Cov}[Q, q_0, s, t] \approx \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \overline{Q})_s (X_i - \overline{Q})_t
\]

\[
M_3[Q, q_0] \approx \frac{1}{n} \sum_{i=1}^{n} \left( \frac{X_i - \overline{Q}}{\sigma} \right)^3
\]

**IV. SIMULATIONS**

Here, we first provide an illustration of the performance of empirical estimators of mean, variance and skewness on end-effector position and showcase the differences in noise profiles that can occur even for simple planar linkages. Next, we consider sampling based approximation of grasp quality for a 7-DOF Kuka Lightweight (LWR) robot arm.

**A. Empirical estimation of noise on end-effector position**

We start by considering the simple planar 2 and 3 link arms depicted in Figure 2. We apply zero mean Gaussian noise with variance of \( \sigma^2 = 0.0175^2 \) and display two nominal arm positions with identical end-effector pose \((x, y) = (2, 2)\), as well as a visualization of the end-effector position for 10,000 samples from the noise distribution.

The distributions of the end effector positions differ not only in spatial orientation, but also in spread, with their empirical covariances \( \Sigma_1, \Sigma_2 \) differing in Frobenius norm by \( 0.000437 \) for the two link arm \( 0.0014 \) for the three links arm. To better understand the convergence of these sampling based estimators for the end-effector position, we sampled 10,000 samples from \( \mathcal{N}(0, \sigma^2) \) for \( \sigma^2 \in \{0.0033^2, 0.0133^2, 0.0233^2, 0.0333^2\} \) in 10 trials. The top panel in Figure 3 displays the convergence of the mean of these 10 runs for the estimators mean, covariance and skewness to a ‘ground truth’ given by the mean estimate of 100,000 samples. As we can observe, the number of required samples to obtain satisfactory convergence is heavily depend on the variance of the noise profile. The bottom panel in the same figure displays the same experiment for the Kuka LWR arm. The charts show that despite the different number of degrees of freedom, 10,000 samples seem sufficient to numerically determine the empirical estimators.

**B. Grasp Quality and Noise**

We next study the dependence of grasp quality on nominal grasp configuration under noise. After describing our experimental setup, we consider the convergence of sampling based grasp quality estimators followed by experiments illustrating interesting cases of the dependence of probabilistic grasp success on variance, object shape and joint configuration.

1) Experimental setup: We use VRep [20] to simulate a 7 degree of freedom lightweight KUKA arm with a Schunk Dexterous hand with three fingers and 7 DOF displayed in Fig. 1. We determined a nominal pre-grasp joint configuration using a grasp planner we developed following the same ideas used in GraspIT! [14], i.e., we sample hand poses around the object to be grasped and through physical simulation we determine if the grasp obtained closing the fingers from the corresponding pose results in a force closure grasp. If this is the case the grasp configuration is retained, otherwise it is discarded. Figure 4 illustrates this process. All computations are performed using VRrep’s built in features for collision detection, and the method eventually returns a set of pre-grasp configurations that will give force closure grasps.

At run time, to generate random samples for a nominal joint-configuration, we added isotropic Gaussian Noise \( \mathcal{N}(0, \sigma^2) \) to each joint of the robot arm and executed the auto-close procedure. Due to noise, the grasp may fail for various reasons. First, the fingers may miss the object during the auto-close step, and therefore not enough contact points can be established to restrain the object. In addition, there are cases in which all fingers make contact with the object, but the resulting placements still do not yield a force closure configuration. Both these instances will be indicated as failures in the following. If instead all fingers establish contact with the object and these points indeed yield force closure, we compute the Ferrari-Canny grasp quality measure. For our experiments, we used the objects displayed in Figure 5.
2) Empirical estimation of noise on grasp quality and probability of force closure: Paralleling the experiments we presented in section IV-A, we start by assessing the impact of noise on grasp effectiveness. In particular, we empirically estimate the probability of force closure, i.e., the probability of obtaining a successful grasp, and the grasp quality metric when a successful grasp is established. Figure 6 shows the empirical estimation for the probability of force closure $P(FC)$ (bottom two charts) and the grasp quality metric (top two charts). The curves were obtained averaging five different grasps for the spray flask object and show a slower convergence rate when compared with Figure 3. This is true for both the grasp quality metric $Q$ and $P(FC)$.

3) Impact of arm configuration: One of the limitations of current grasp metrics is that they do not consider the arm configuration used to implement a target grasp. However, when noise is explicitly modeled, significant differences may emerge when comparing different arm configurations achieving the same contact points. To study this effect, we start by considering two objects grasped with the same contact points but different arm configurations. Figure 7 shows the two objects and the variability for the grasp. Colored sticks are used to display the normal to the object surface at each contact point, with different colors used for the three fingers. The figures display the results obtained with 10,000 samples. For the drill object we determine that the mean grasp quality metric is comparable in the two cases (0.0982 versus 0.0910) and the variance is comparable too (0.0025 versus 0.0028). However, there is a remarkable difference in the probability of force closure $P(FC)$. The left configuration achieves a value of $P(FC) = 0.50$, whereas the right configuration has a value $P(FC) = 0.31$. Similar observations are made for spray flask, where the right configuration has $P(FC) = 0.57$ and the left one has $P(FC) = 0.42$.

4) Impact of noise on grasp quality: Finally, we extensively evaluate the impact of noise on $P(FC)$ and the grasp quality metric for different objects, grasps, and noise levels. We start considering the spray flask object with five different grasps and four different noise levels. Figure 8 shows the distribution of normals to the contact points using the same

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1Note that since $P(FC)$ is the probability of a binary indicator random variable $FC$, the mean of $FC$ is equal to $P(FC)$.
Fig. 6: Empirical estimation of the convergence rate for the estimators of the Ferrari-Canny grasp quality metric (top charts) and probability of force closure (bottom charts).

coloring used in Figure 7. Each row corresponds to a different grasp, and each column to a different variance in the joints, i.e., 0, 0.0033, 0.0133, 0.0233, 0.0333 (left to right). Table 9 displays the numerical results for these grasps, i.e., probability of force closure \( P(FC) \) and the mean and variance of the Ferrari-Canny grasp quality metric for the successful grasps. Results are obtained using 10,000 samples. From the table we can observe that \( P(FC) \) appears to be more sensitive to variations in the joint noise, as measured in terms of the variance \( \sigma^2 \). Grasp quality varies too, although the variation is more modest and at times even non monotonic.

As last experiment, for the same four objects shown in Figure 5 we consider ten different grasps and evaluate the effect of noise in terms of \( P(FC) \) and grasp quality for the successful grasps. Results are displayed in Figures 9 and 10. Figure 9 displays the results for grasps configurations consisting of different contact points and different arm configurations. Figure 10 instead considers same contact points but different arm configurations due to different placements of the objects with respect to the arm. In both cases joint angles were affected by Gaussian noise \( N(0, 0.0033^2) \). As it can be seen, generally speaking \( P(FC) \) is uniformly high for a simple object like the cube, while is may dramatically vary for more complex objects. Quality measure is the average value for the Ferrari-Canny metric limited to the grasps for which force closure is obtained. In the two figures we can see more variations for the first case where variations involve both the contact points and the arm configuration. Finally

**Error** is the difference between the theoretical grasp quality metric predicted by the planner without considering noise sources, and the average grasp quality metric experimentally determined. As it can be seen, this value is almost always positive, indicating that in most cases noise alters the contact points in a way that negatively impacts the quality measure, although in some sporadic this observation is not valid, i.e., noise actually ends up generating contact points with a better quality (and then error values are negative.)

V. CONCLUSIONS AND FUTURE WORK

In this paper we have formulated a framework to study the interplay between grasp quality evaluation functions, noise, and the mechanical structure of the robotic arm. This work fills a gap in the grasp evaluation literature inasmuch as grasp quality metric studies have ignored these aspects and have mostly evaluated grasps seen as a set of deterministic contact points. Unsurprisingly, our experiments show that there is a dependency, but the notable aspect is that this can be significant.

This work lies the foundation for a number of follow up studies. In particular, we believe that a fundamental re-thinking of grasp quality metrics is needed to account for both the noise and the robot structure. The proposed method based on sampling outlines the problem, but is not practical in an online scenario. This is strongly dependent on the algorithm used to compute the wrench metric and the number of samples and can currently be in the order of few minutes. To this end, it would be interesting to determine grasp quality metrics that can be expressed as an analytic function of the contact points, so that its Jacobians could be explicitly computed and plugged into an analytic approximations.

Finally, in this paper we have mostly concentrated on the noise affecting the robot joints and how it impacts the pose of the end effector and then the success rate of a grasp or its quality. One could argue that in some robots this noise may be negligible, though the recent advent of low cost
TABLE I: Probability of force closure, mean and variance of the grasp quality metric for the grasps in figure 8. Grasp quality metric is computed only for successful grasps.

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REFERENCES


Fig. 9: Success rate, quality measure, and error for ten different grasps with different contact points and different arm configuration.

Fig. 10: Success rate, quality measure, and error for ten different grasps with same contact points but different arm configuration.
